B. Sc. Part III (7th Paper)

Diff. equations

Some sums on standard form II

$$\boxed{z = px + qy + f(p,q)}$$

I Solve $(p+q)(z-px-qy) = 1$.

Soln The given equation

$(p+q)(z-px-qy) = 1$

$\Rightarrow \quad z - px - qy = \dfrac{1}{p+q}$

$\Rightarrow \quad z = px + qy + \dfrac{1}{p+q}$ ——(1)

which is of the form

$z = px + qy + f(p,q)$.

Its complete integral is given by

$z = ax + by + \dfrac{1}{a+b}$ ——(2)

where a and b are two constants.

Let $b = \phi(a)$

$\Rightarrow \quad z = ax + y\phi(a) + \dfrac{1}{a+\phi(a)}$ ——(3)

Differentiating it partially w.r.to a, we get

$0 = x + y\phi'(a) - \dfrac{1}{[a+\phi(a)]^2}(1+\phi'(a))$ ——(4)

Elimination of a from (3) and (4) gives the general integral of the given equation.

From (2), $z = ax + by + \frac{1}{a+b}$ —— (2)

Diff it partially w.r.to a, we get

$0 = x - \frac{1}{(a+b)^2}$ —— (3)

Diff (2) partially w.r.to b, we get

$0 = y - \frac{1}{(a+b)^2}$ —— (4)

From (3), $x = \frac{1}{(a+b)^2}$

From (4) $y = \frac{1}{(a+b)^2}$

From (2) $\Rightarrow z = a \times \frac{1}{(a+b)^2} + \frac{b}{(a+b)^2} + \frac{1}{(a+b)}$

$= \frac{a+b}{(a+b)^2} + \frac{1}{(a+b)}$

$= \frac{2}{(a+b)}$

$\Rightarrow z = \frac{2}{\cancel{\sqrt{x}}}\sqrt{x} = 2\sqrt{y}$

This is the ~~c~~ sing. int.

2. Find the integrals of

$$z = px + qy + \log pq.$$

Soln

The given eqn

$$z = px + qy + \log pq \quad \text{---} (1)$$

It is of the form

$$z = px + qy + f(p,q)$$

$\therefore$ complete integral is given by

$$z = ax + by + \log(ab) \quad \text{---} (2)$$

Put $b = \phi(a)$

$$\Rightarrow z = ax + y\,\phi(a) + \log\{a\,\phi(a)\} \quad \text{---} (3)$$

Diff. it partially w.r.to a, we get

$$0 = x + y\,\phi'(a) + \frac{1}{a\,\phi(a)}\left[\phi(a) + a\,\phi'(a)\right] \quad \text{---} (4)$$

Elimination of a from (3) and (4) gives the general integral.

(2) is $\quad z = ax + by + \log ab$

$$\Rightarrow z = ax + by + \log a + \log b \quad \text{---} (5)$$

Diff. it partially w.r.to a, we get

$$0 = x + \frac{1}{a} \Rightarrow a = -\frac{1}{x} \quad \text{---} (6)$$

Differentiating (5) partially w.r.to b, we get

$$0 = y + \frac{1}{b} \Rightarrow b = -\frac{1}{y} \quad \text{——} (7)$$

Putting the values of a and b from (6) and (7) in (2) we get

$$z = ax + by + \log(ab)$$

$$\Rightarrow z = x \times -\frac{1}{x} + y \times \left(-\frac{1}{y}\right) + \log\left(-\frac{1}{x} \times -\frac{1}{y}\right)$$

$$= -1 - 1 + \log\left(\frac{1}{xy}\right)$$

$$= -2 - \log xy$$

$$\Rightarrow z = -(2 + \log xy)$$

This is the singular integral.